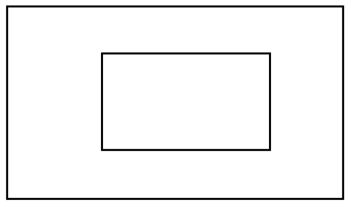
Lesson: How do changes in dimensions of objects affect their perimeter, area and volume?

Eighth Grade Objective: **2.01** Determine the effect on perimeter, area or volume when one or more dimensions of two- and three-dimensional figures are changed.

Lesson:
A photographer is framing her latest print. Her print is 5 inches by 7 inches and her frame is twice as long and twice as wide as the photograph. Looking at the picture, she believes that she could actually fit four photographs into the frame, instead of just the one. How can this be when each dimension was only doubled?



Remember, the original photograph was 5 inches by 7 inches, which means it has an area of 35 square inches (length times width). The dimensions of the frame are twice the dimensions of the photograph, or 10 inches by 14 inches. The area of the frame must be 10 x 14 = 140 square inches.

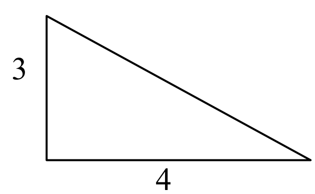
Four photographs would fit into the frame since 140 / 35 = 4. When the side lengths were doubled, the area increased by a factor of four.

This can be done with algebra, also. The formula for area of a rectangle is lw. If each dimensions is doubled, it can be written as (2l)(2w) which equals 4lw. The area went from lw to 4lw, it changed by a factor of four.

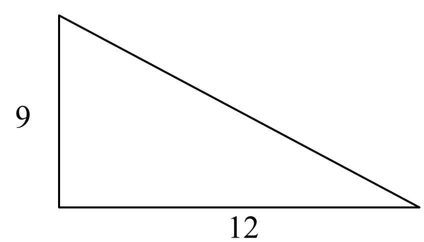
Let's try:

Using either numerical relationships or algebraic reasoning, determine the factor by which each area increases (images are NOT drawn to scale).

1. Original:



New:

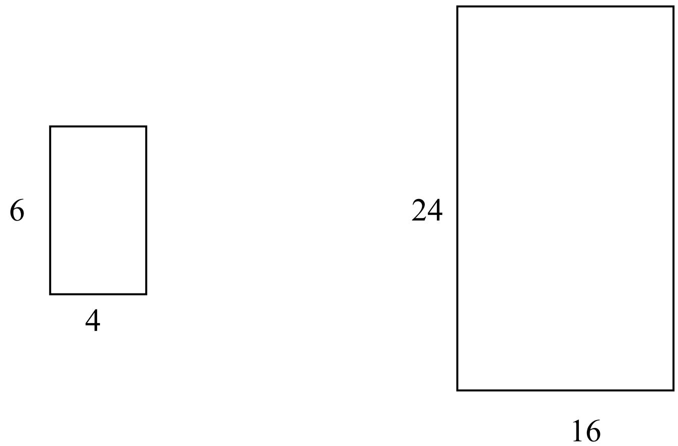


The old area is: ½ bh or ½ * 3 * 4 = 6 units squared.
The new area is: ½ bh or ½ * 9 * 12 = 54 units squared
The area increased by a factor of 54/6 or 9.

Using algebra: we notice that each dimension has increased by a scale factor of 3.
Area of the original = ½ bh

Area of the new = ½ (3b)(3h) or ½ (9)bh.
The area increased by a factor of 9.

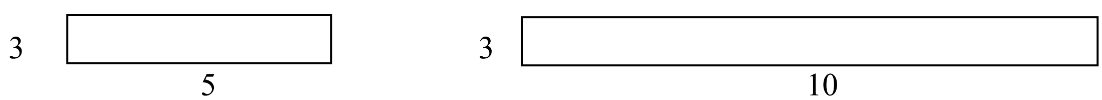
2.



The old area is: lw or 6 * 4 = 24 square units.
The new area is: lw or 24 * 16 = 384 square units.
The area increased by a factor of 384/24 or 16.

Using algebra: we notice that each dimension has increased by a scale factor of 4.
Area of the original = lw
Area of the new = (4l)(4w) or 16lw.
The area increased by a factor of 16.

3.



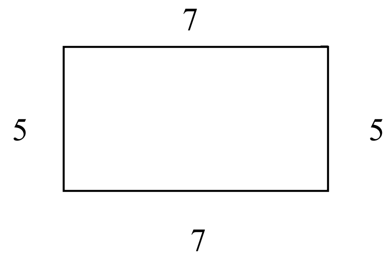
The old area is: lw or 3 * 5 = 15 square units.
The new area is: lw or 3 * 10 = 30 square units.
The area increased by a factor of 30/15 or 2.

Using algebra: we notice that only the width has increased by a scale factor of 2.
Area of the original = lw
Area of the new = (l)(2w) or 2lw.
The area increased by a factor of 2.

We've looked at area, now let's look at what happens to perimeter when dimensions change.

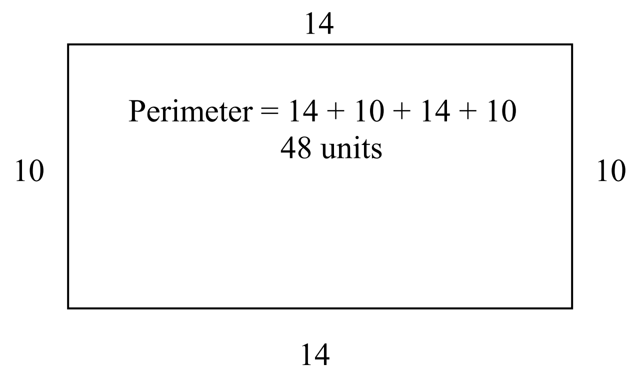
Back to our photographer: In order to save money on framing supplies, she has decided that maybe she is just going to make a frame that goes directly around the perimeter of the 5x7 photograph, instead of using a 10 x 14 frame. Since she found out that the larger frame would actually hold four pictures, she thinks that the cost for the wood to frame the picture would be four times as expensive. Let's see…

How much wood does she need to make a 5x7 frame? We are referring to the distance around a shape, so we need to find the perimeter.



perimeter= 5+7+5+7
24 units

Now, how much wood does she need to make a 10x14 frame?



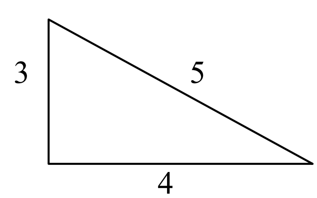
The perimeter changed from 24 to 48. 48/24 is 2, so the perimeter changed by a scale factor of 2. The wood for the larger frame would be twice as expensive as for the smaller frame.

We could also look at this algebraically:
The perimeter of the original is $l + l + w + w = 2l + 2w$
The perimeter of the new is $2l + 2l + 2w + 2w = 4l + 4w$ or $2(2l + 2w)$, the perimeter increased by a factor of 2.

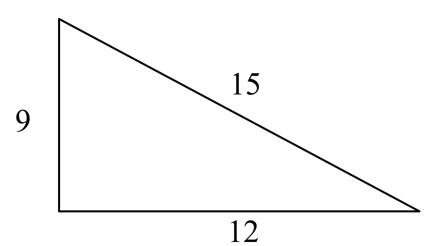
Let's try:

Using either numerical relationships or algebraic reasoning, determine the factor by which each perimeter increases (images are NOT drawn to scale).

1. Original:



New:

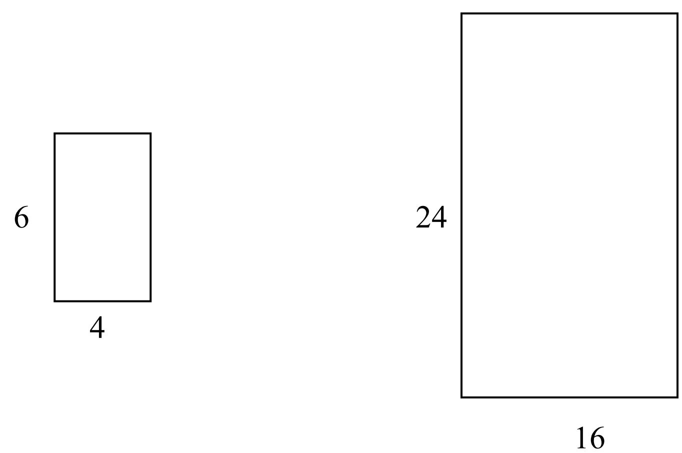


The old perimeter is: 3 + 4 + 5 = 12 units.

The new perimeter is 9 + 12 + 15 = 36 units squared
The perimeter increased by a factor of 36/12 or 3.

Using algebra: we notice that each dimension has increased by a scale factor of 3.
Perimeter of the original = b + h + d
Perimeter of the new = 3b + 3h + 3d = 3(b + h + d)
The perimeter increased by a factor of 3.

2.



The old perimeter is: 4 + 6 + 4 + 6 = 20 units.
The new perimeter is: 24 + 16 + 24 + 16 = 80 units.
The perimeter increased by a factor of 80/20 or 4.

Using algebra: we notice that each dimension has increased by a scale factor of 4.
Perimeter of the original = l + l + w + w = 2l + 2w
Perimeter of the new = 4l + 4l + 4w + 4w = 8l + 8w = 4(2l+2w).
The perimeter increased by a factor of 4.

We only have one more relationship to determine. What happens to the volume of an object as the dimensions change?

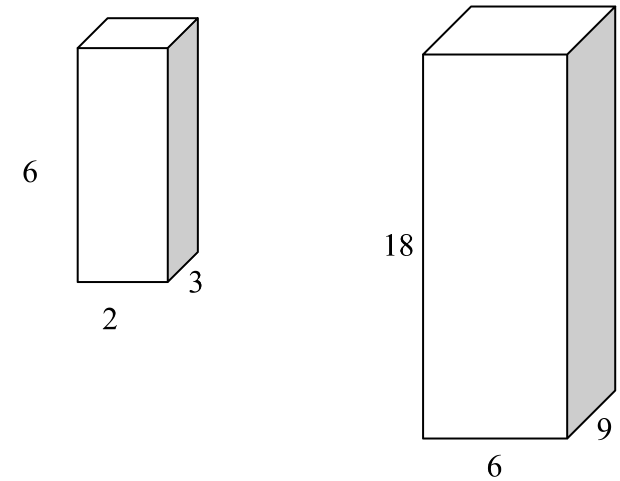
Let's say our photographer needs to store the chemicals that she uses to develop prints at home. She has two containers full of liquid that have the following dimensions: length 3 inches, width 4 inches and height 5 inches. She has one container that has double each of the smaller containers dimensions. Will she be able to fit all the chemicals into the larger container, and if so, how much room is left over?

To find the volume of the original container: v = lwh or 3 x 4 x 5 = 60 cubic inches.
To find the volume of the larger container: v = (2l)(2w)(2h) = 8lwh or 6 x 8 x 10 = 480 cubic inches.
She can fit both of her smaller containers into the larger container with 420 cubic inches of space left over! She could fit 480/60 = 8 little containers into the larger container.

Let's try these:

Determine how many smaller containers the larger container can hold (in other words, find the factor by which the volume has increased).

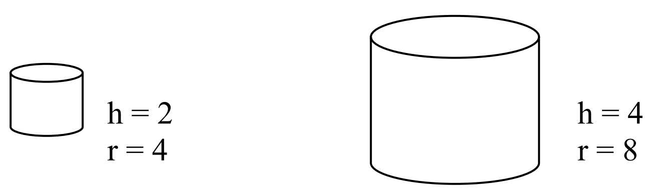
1.



Volume of the original = 6 x 2 x 3 = 36 cubic units
Volume of the new = 18 x 6 x 9 = 972 cubic units
972/36 = 27. Twenty-seven small containers can fit in the larger container.

Old volume: V = lwh
Each dimension was tripled new volume = (3l)(3w)(3h) = 27lwh.
The volume increased by a factor of 27.

2.



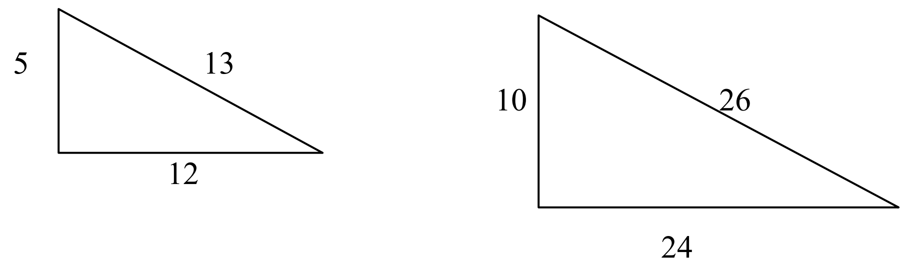
Volume of the original = $\prod r^2 h = \prod (4)^2 2$ = 100.5 units cubed
Volume of the new = $\prod r^2 h = \prod (8)^2 4$ = 804.2 units cubed
804.2/100.5 = approximately 8 (we rounded our first two answers, which is why this isn't exact)

Old volume = $\prod r^2 h$
Each dimension was doubled: new volume = $\prod (2r)^2 (2h) = \prod * 4r^2 * 2h = 8\prod r^2 h$
The volume increased by a factor of 8.

Try these on your own:

Find the factor by which the perimeter and the area have increased.

1.



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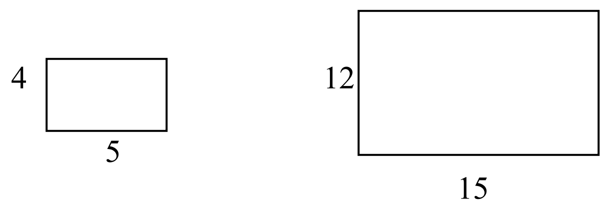
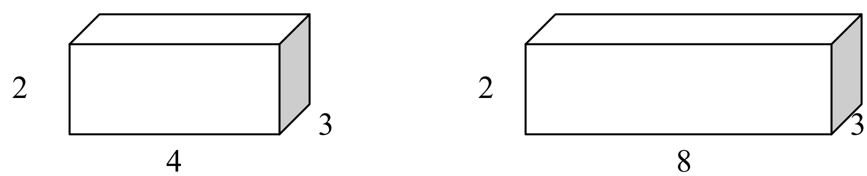
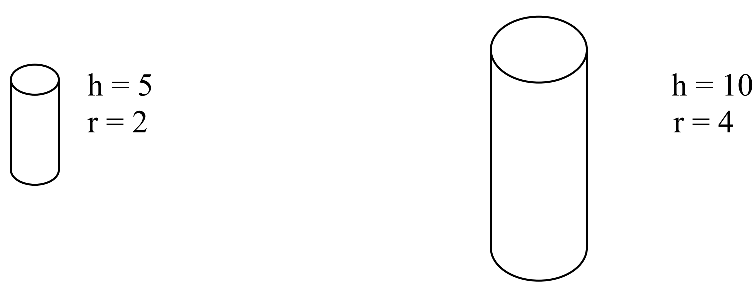


Determine by which factor the volume increased (how many of the smaller shapes will fit in the larger shape).

3.



4.

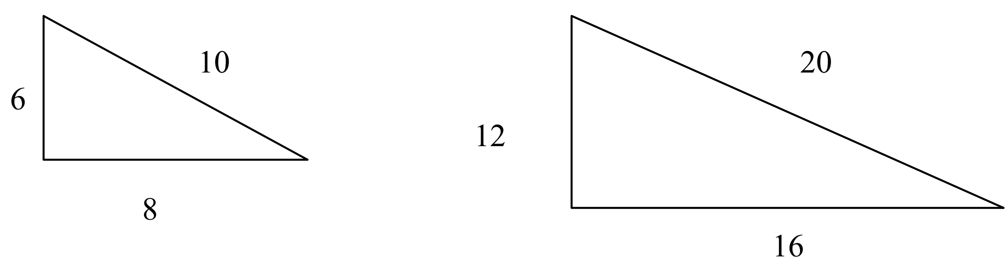


Check your answers:

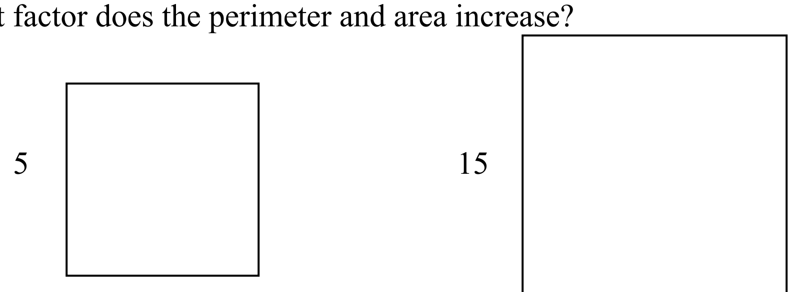
1. The perimeter of the first figure is 5 + 12 + 13 = 30, the perimeter of the second is 10 + 24 + 26 = 60. The perimeter doubled.
   The area of the first figure is ½ 5 * 12 = 30, the area of the second figure is ½ 10 * 24 = 120. The area increased by a factor of 4. [a = ½ bh becomes a = ½ (2b)(2h) = ½ 4bh]
2. The perimeter of the first figure is 4 + 5 + 4 + 5 = 18, the perimeter of the second is 12 + 15 + 12 + 15 = 54. The perimeter increased by a factor of three.
   The area of the first figure is 4x5 = 20. The area of the second figure is 12x15 = 180. The area increased by a factor of 9. [a = lw becomes a = (3l)(3w) = 9lw]
3. The volume of the original is 2x3x4 = 24 cubic units. The volume of the new figure is 2x3x8 = 48 cubic units. The volume increased by a factor of 2. [V = lwh becomes V = l(2w)h or 2lwh]
4. The volume of the original is approximately 62.8 units cubed, 502.7 units cubed. The volume increased by a factor of 8. [$V = \prod r^2 h$ becomes $\prod(2r)^2 h = \prod 4r^2 h$]

Quiz yourself:

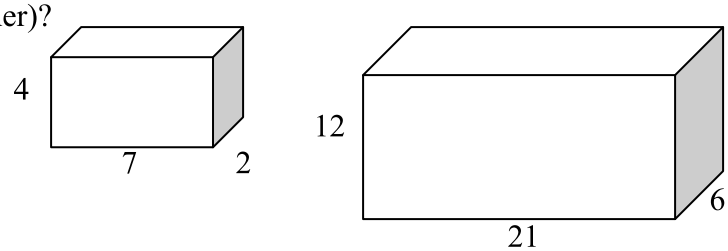
1. By what factor does the perimeter and area increase?



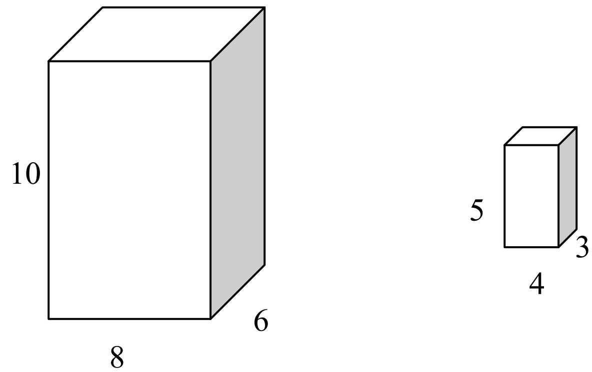
2. By what factor does the perimeter and area increase?



3. By what factor does the volume increase (how many small containers can fit in the larger container)?



4. Challenge: By what factor does the volume change (how much of the large container can fit in the small container)?



Check your answers:

1. Perimeter doubles, area quadruples (increased by a factor of four).
2. Perimeter triples, area increases by a factor of nine.
3. Volume increases by a factor of 27 (27 small containers can fit in the larger container).
4. Volume decreases by a factor of 1/8. One-eighth of the large container will fit in the smaller container.