Lesson: Best fit line/curve.

Eighth Grade Objective: **4.02 Approximate a line of best fit for a given scatterplot; explain the meaning of the line as it relates to the problem and make predictions.**

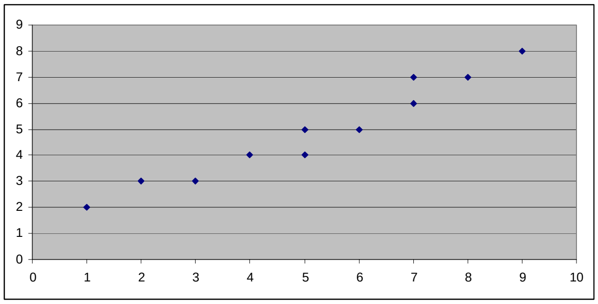
Lesson:

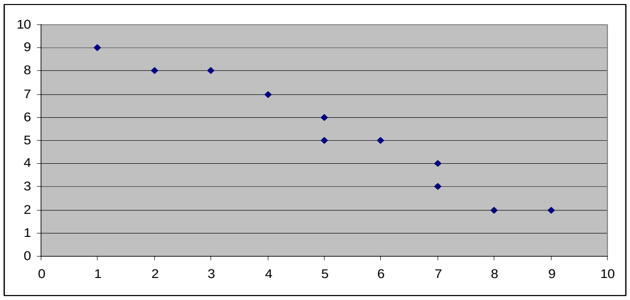
In the lesson for Goal 4.01, we analyzed scatterplots, to make predictions for values that are not given to us. In this lesson, we will write equations for relationships and use those equations to make predictions.

Because scales and intervals vary, when we draw scatterplots, the images may appear slightly different. In the same light, when we draw a line of best fit (or best fit curve), we are approximating the data and our approximations may be slightly different. As we proceed through this lesson, keep in mind the numerical values you come up with might be slightly different that what is suggested here. As long as you are within a close range, your solution may be acceptable.

Before we can make a prediction from a scatterplot, we must first recognize the types of trends commonly seen in data, as well as the types of equations associated with them.

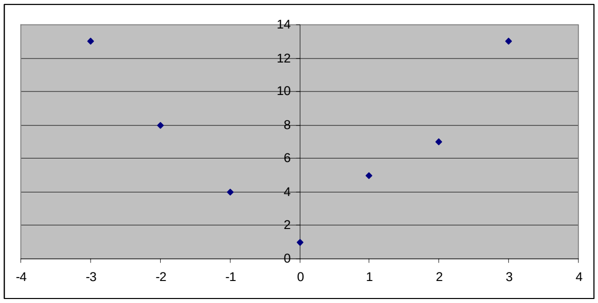
Linear trends are those whose points generally follow the trend of a line. The line can have a positive (upward) trend, or a negative (downward) trend.

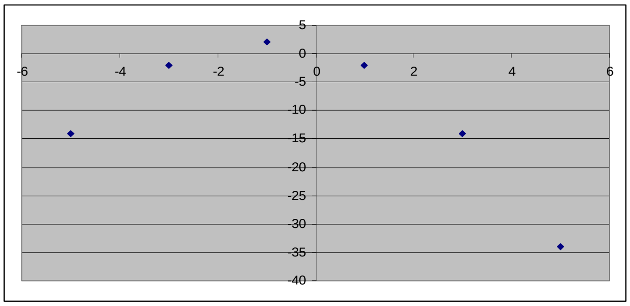




Linear equations are written in the form y = mx + b, where m is the slope of the line and b is the y-intercept.

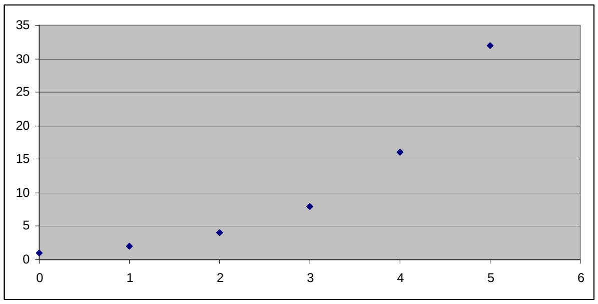
Quadratic trends are those whose points generally follow the trend of a quadratic graph. Quadratic graphs are in the shape of a parabola, or a U-shape.

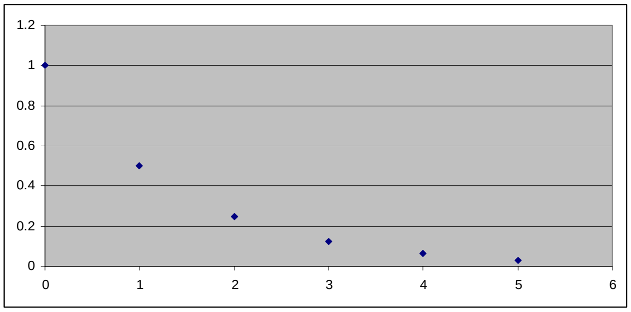




Quadratic equations are written in the form $y = Ax^2 + Bx + C$. We will use the roots, or zeros, and a little algebra to determine equations for these types of curves.

Exponential graphs are those whose points generally follow the trend of an exponential graph. Exponential graphs can start out increasing slowly and then begin to increase rapidly (exponential growth) or they can decline rapidly and then begin to decline more slowly (exponential decay).





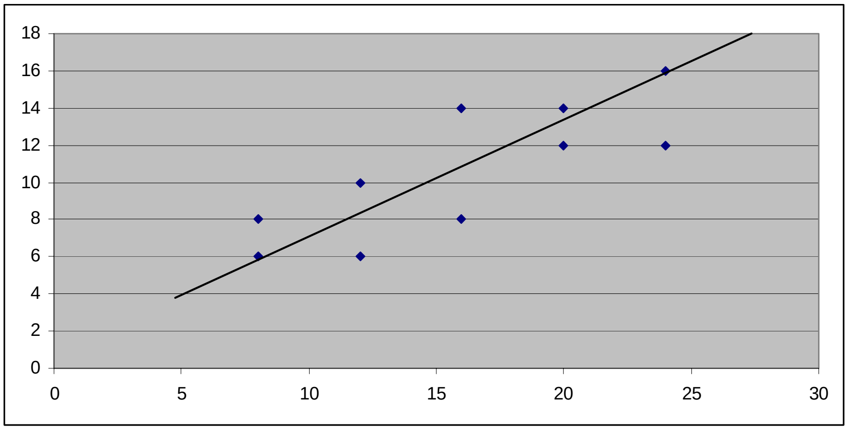
Exponential equations are written in the form $y = a * b^x$, where a is the starting point (y-intercepts) and b is the growth factor.

Graph a scatterplot of the data values given. Sketch a curve of best fit and write an equation for the curve. Use the equation to make the predictions indicated.

x	y
8	8
8	6
12	6
12	10
16	8
16	14
20	14
20	12
24	16
24	12

Predict the value of y when x = 4.

Predict the value of x when y = 20.



The graph appears to have a positive linear relationship. Two points on the line include (8, 6) and (24,16).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{16 - 6}{24 - 8}$$

$$m = 10/16 = 5/8$$

We can use the slope and one of the points to determine the equation for the line:

$$y = mx + b$$

$$16 = (5/8)(24) + b$$

$$16 = 15 + b$$

$$1 = b$$

Now we can write the equation using the slope and y-intercept:

$$y = mx + b$$

$$y = (5/8)x + 1$$

Now we use our equation to make our predictions:

When $x = 4$: $y = (5/8)*4 + 1$

$$y = 3.5$$

$$(4, 3.5)$$

When $y = 20$: $20 = (5/8)x + 1$

$$19 = (5/8)x$$

$$30.4 = x$$

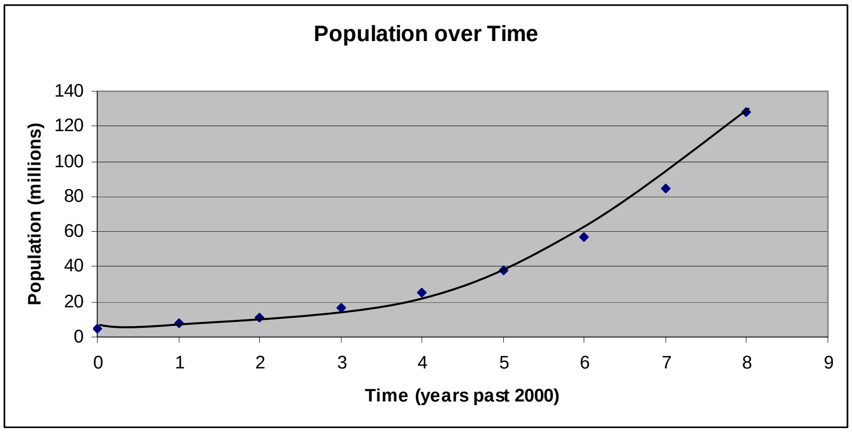
$$(30.4, 20)$$

Remember, if your equation and predicted values are CLOSE, you are most likely correct. If your equation is slightly different, of course your predicted values will be different.

Years past 2000	Population (millions)
0	5
1	8
2	11
3	16
4	25
5	38
6	57
7	85
8	128

Predict the year in which the population will exceed 500 million.

Prediction the population in the year 2010.



The data appears to show exponential growth. To write an equation for exponential growth, we need the y-intercept of the graph (which is (0, 5)) and the growth factor. In this case, the growth factor is most easily determined by looking at the table. Let's look at the multiplicative change in y for each unit change in x:

5 * b = 8; b = 1.60
8 * b = 11; b = 1.38
11 * b = 16; b = 1.45
16 * b = 25; b = 1.56
25 * b = 38; b = 1.52
38 * b = 57; b = 1.5
57 * b = 85; b = 1.49
85 * b = 128; b = 1.51

Now let's average these changes: 1.5

Write the equation of the curve of best fit: $y = a * b^x$
$y = 5 * 1.5^x$

Now we use our equation to make our predictions:

When $y = 500$: $500 = 5 * 1.5^x$

$$100 = 1.5^x$$

Although there is a strategy for “undoing” exponents, at this point, we do not know how to do that, so we will guess and test: 1.5 raised to the x power = 100. I first put 20 in for x and the result was WAY too high (3325). I then tried 15 and the answer was still way too high (438). Then I tried 11 and it was close, but a little low (86.5), so I tried 12 and it was high (130). So our answer is 2012 – the question asked when the population would exceed the given population. It will exceed it in 2012. (The exact answer for x is 11.36, but again, it is not expected that you would arrive at such a specific prediction).

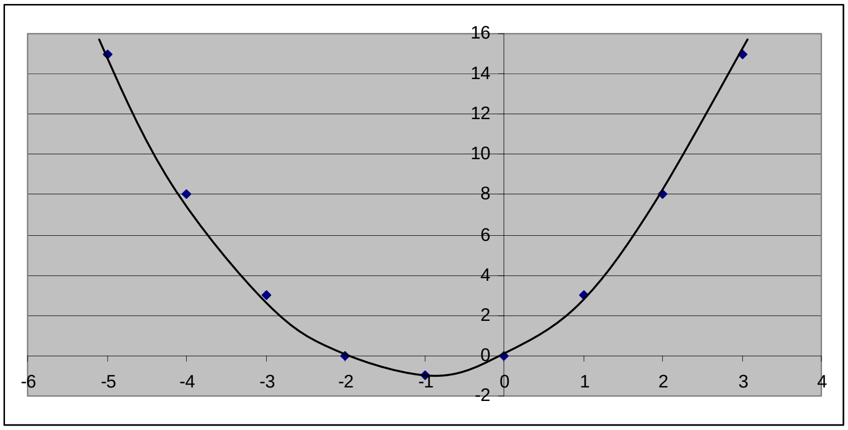
When $x = 10$ (remember that x is the number of years past 2000):

$$y = 5 * 1.5^{10}$$

$$y = 288.3 \text{ million}$$

x	y
-5	15
-4	8
-3	3
-2	0
-1	-1
0	0
1	3
2	8
3	15

Predict the value for y when x is 7.



To write an equation for a quadratic, we will focus on the x-intercepts: (0, 0) and (-2, 0).
Another way to write the ordered pair (-2, 0): $x = -2$
Which can be rewritten as: $x + 2 = 0$

AND

Another way to write the ordered pair (0, 0): $x = 0$

Now we can take our two green equations and write them together as multiplication (this will be explained more in depth in algebra I):

$$(x + 2)(x) = y$$
$$x^2 + 2x = y$$

We can use this equation to make our predictions:
When $x = 7$:

$$7^2 + 2(7) = y$$
$$49 + 14 = y$$
$$y = 63$$

Try these on your own!

Draw a scatterplot and curve of best fit for each data set. Write an equation for the curve and make the indicated predictions.

1.

Temperature (degrees F)	Heating Costs
0	70
10	60
20	53
30	45
40	35
50	28
60	20
70	8
80	0

Predict the heating cost when it is 47 degrees.

Predict the temperature when the heating costs are $90.

2.

Time (days)	Amount of radioactive element remaining (grams)
0	200
1	100
2	50
3	25
4	13
5	6
6	3

Predict the amount of the radioactive element that exists after 8 days.

3.

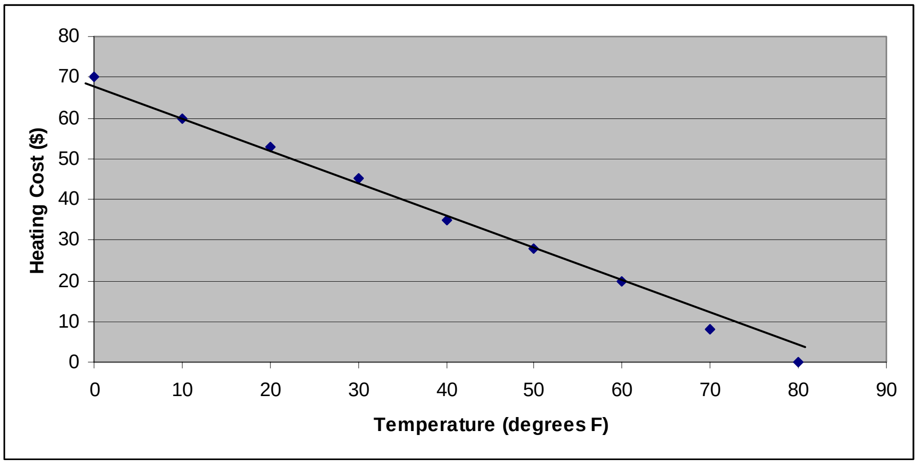
Side length of a rectangle with perimeter 24 (inches)	Area of the rectangle (square inches)
1	11
2	20
3	27
4	32
5	35
6	36
7	35
8	32
9	27
10	20
11	11

Predict the area of the rectangle if the side length is 5.5 inches.

Check your answers:

1.

Heading Cost v Temperature



The graph appears to have a negative linear relationship. Two points on the line include (10, 60) and (60, 20).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{60 - 20}{10 - 60}$$

$$m = 40/-50 = -4/5$$

We can use the slope and one of the points to determine the equation for the line:

$$y = mx + b$$

$$60 = (-4/5)(10) + b$$

$$60 = -8 + b$$

$$68 = b$$

Now we can write the equation using the slope and y-intercept:

$$y = mx + b$$

$$y = (-4/5)x + 68$$

Now we use our equation to make our predictions:

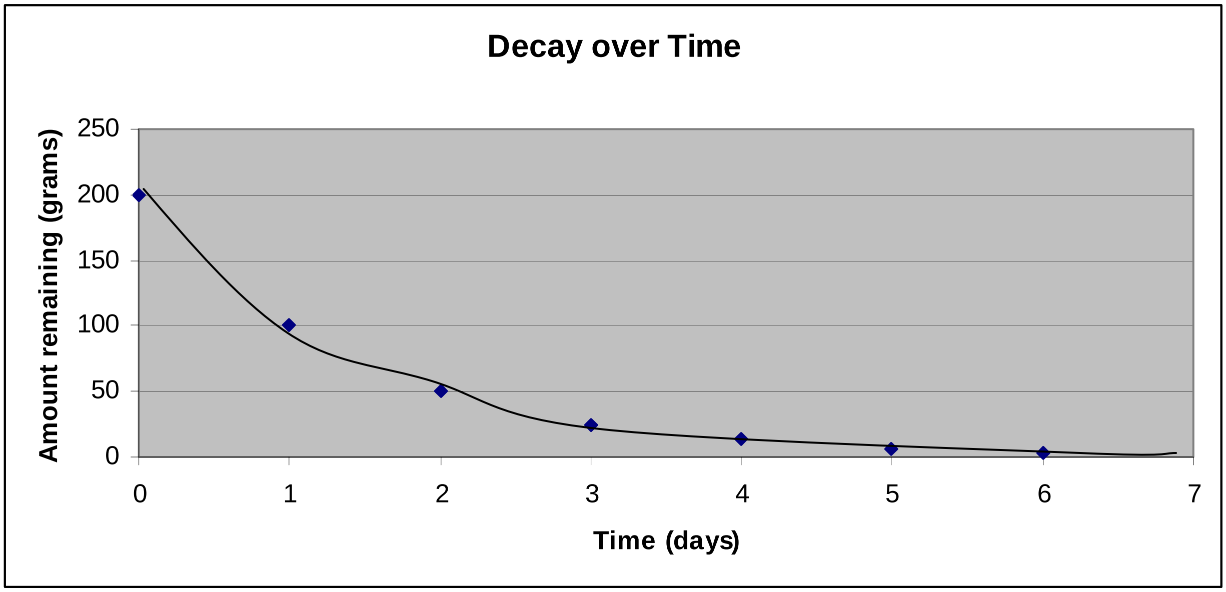
When x = 47: $y = (-4/5)*47 + 68$

$$y = 30.4$$

When y = 90: $90 = (-4/5)x + 68$

$$22 = (-4/5)x$$
$$-27.5 = x$$

2.



The data appears to show exponential growth. To write an equation for exponential growth, we need the y-intercept of the graph (which is (0, 200)) and the growth factor. In this case, the growth factor is most easily determined by looking at the table. Let's look at the multiplicative change in y for each unit change in x:

200 * b = 100; b = ½
100 * b = 50; b = ½
50 * b = 25; b = ½
25 * b = 13; b = 0.52
13 * b = 6; b = 0.46
6 * b = 3; b = ½

Now let's average these changes: approximately ½

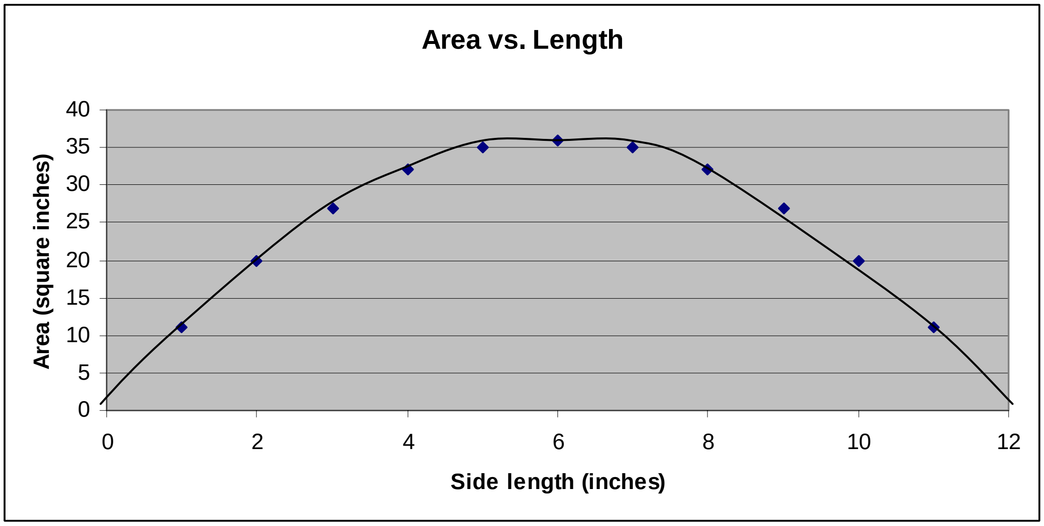
Write the equation of the curve of best fit: $y = a * b^x$
$$y = 200 * 0.5^x$$

Now we use our equation to make our predictions:

After 8 days have passed (x = 8) $y = 200 * 0.5^8$

$y = 0.78125$

3.



To write an equation for a quadratic, we will focus on the x-intercepts: (0, 0) and (12, 0) – note the symmetry in the graph.

Another way to write the ordered pair (12, 0): $x = 12$

Which can be rewritten as: $-x + 12 = 0$

AND

Another way to write the ordered pair (0, 0): $x = 0$

Now we can take our two green equations and write them together as multiplication (this will be explained more in depth in algebra I):

$$(-x + 12)(x) = y$$
$$-x^2 + 12x = y$$

We can use this equation to make our predictions:

When x = 5.5: $-x^2 + 12x = y$

$$-(5.5)^2 + 12(5.5) = y$$

$y = 35.75$ inches squared

Quiz yourself!

Draw a scatterplot and curve of best fit for each data set. Write an equation for the curve and make the indicated predictions.

1.

x	y
5	12
7	15
9	17
11	20
13	22
15	25
17	27

Predict the x value when y is 40.

Predict the y value when x is 40.

2.

x	y
-3	0
-2	-2
-1	-2
0	0
1	4
2	10
3	18

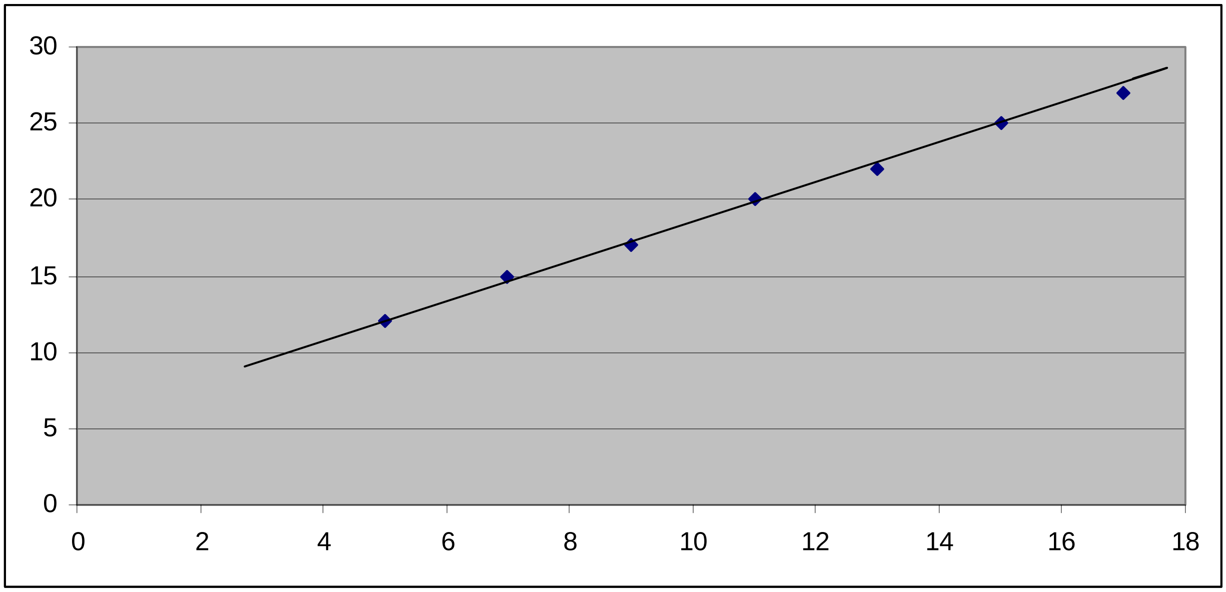
Predict the y value when x is 20.

3.

x	y
0	10
1	29
2	87
3	275
4	805
5	2432
6	7284

Predict the y value when x is 3.5.

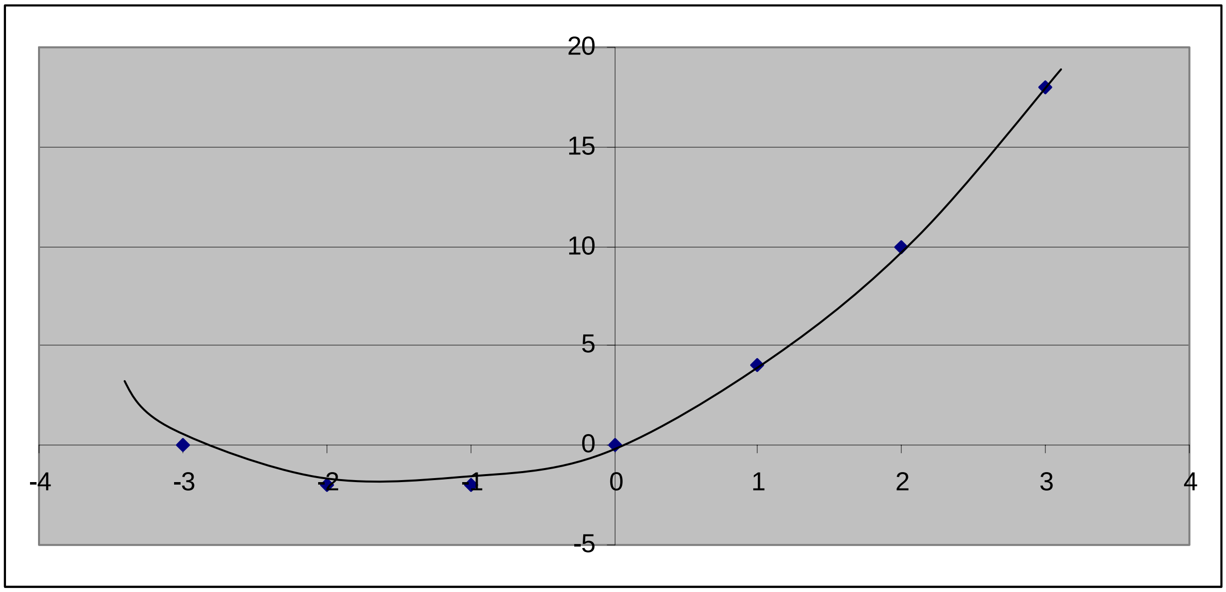
Check your answers:
1.



An appropriate equation for the line shown is: y = 1.3x + 5.5.
The ordered pairs (5, 12) and (15, 25) were used to arrive at this equation.

When x is 40, y is 57.5.
When y is 40, x is 26.5.

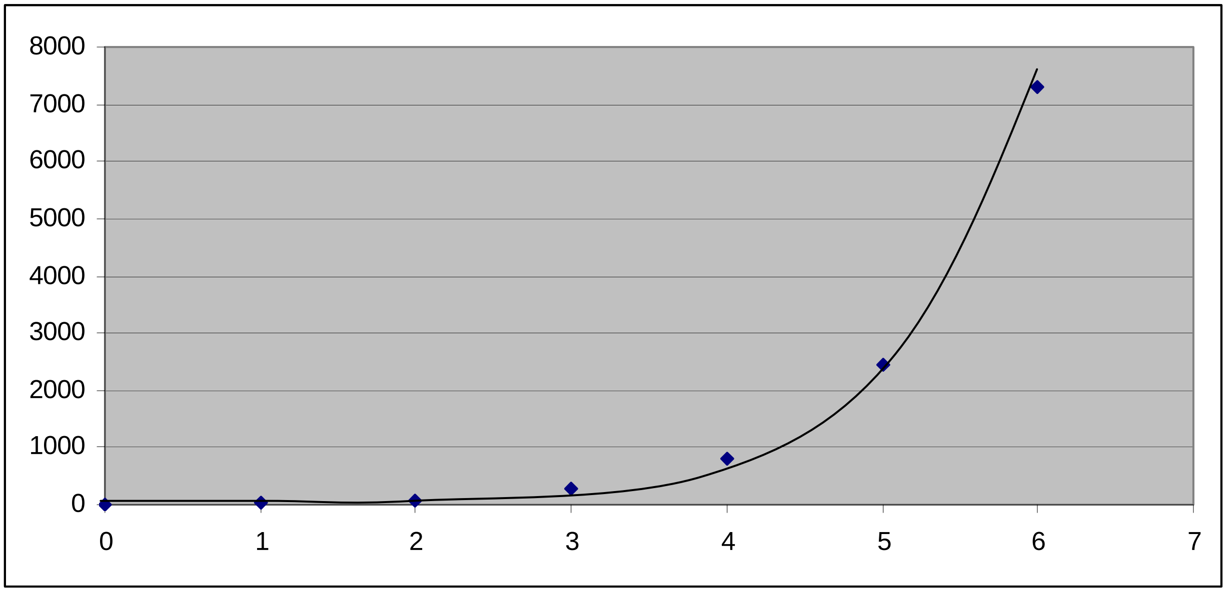
2.



An appropriate equation to use for this quadratic graph uses the zeros (-3, 0) and (0, 0), or the equations $x + 3 = 0$ and $x = 0$. When multiplied, those become the equation: $y = x^2 + 3x$.

When x is 20, y is 460.

3.



An equation that can be used to model this exponential has starting point (0, 10) and an average multiplicative change of 3. $y = 10 * 3^x$. The starting point is most easily read off the chart, since the graph has such a large scale.

When x is 3.5, y is 467.7.